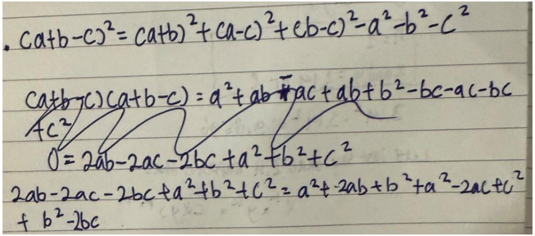
MTH 1020 Week 5 tutorial

1. Broad feedback on Ass 1 to start with

General feedback, assignment 1

- In future if you write your assignments by hand please *scan* rather than taking a photo (there are PDF scanning apps for the phone too).
- **Please submit a PDF, not a Word document.**
- DO NOT USE ABBREVIATIONS AND SYMBOLS LIKE $\implies$ AND $\therefore$ IN THE MIDDLE OF SENTENCES.
- Q1 (ii) many people missed: if you use (i), you do not need to do much more work.
- Q2 (a)
  - simple proof: $x + 1/x - 2 = (x-1)^2/x$; this is positive iff x is positive. Will accept other proofs structured as a direct proof e.g. by cases, so long as they do not use machinery like calculus without proof.
  - Several people used AM/GM inequality. This is not valid when one number is positive and the other number is negative, so you needed additional line to explain why the hypotheses were valid. In addition you should be able to prove AM/GM if required, though I will accept it here since it is not using entire new theory like calculus.
- Q3 Clearly state what the inductive hypothesis is and where you use it.

Assignment 1 feedback: Q1(i)



$(a+b-c)^2 = (a+b)^2 + (a-c)^2 + (b-c)^2 - a^2 - b^2 - c^2$

$(a+b-c)(a+b-c) = a^2 + ab - ac + ab + b^2 - bc - ac - bc + c^2$

$0 = 2ab - 2ac - 2bc + a^2 + b^2 + c^2$

$2ab - 2ac - 2bc + a^2 + b^2 + c^2 = a^2 + 2ab + b^2 + a^2 - 2ac + c^2 + b^2 - 2bc$

This is not a complete proof, this is just some scratch algebra. Think about what you are trying to communicate.

Assignment 1 feedback: Q1(i)

Q1. Prove that for all $a, b, c \in \mathbb{R}$ that

(i) $(a+b-c)^2 = (a+b)^2 + (a-c)^2 + (b-c)^2 - a^2 - b^2 - c^2$

To prove this, we can expand the left hand side and the right hand side separately.

$$LHS = (a+b-c)^2 = (a+b-c)(a+b-c)$$
$$= a^2+b^2+c^2+ab-ac+ba-bc-ac-bc$$
$$= a^2+b^2+c^2+2ab-2bc-2ac$$

$$RHS = (a+b^2)+(a-c)^2+(b-c)^2-a^2-b^2-c^2$$
$$= (a+b)(a+b)+(a-c)(a-c)+(b-c)(b-c)-a^2-b^2-c^2$$
$$= a^2+b^2+c^2+ab+ab-ac-ac-bc-bc$$
$$= a^2+b^2+c^2+2ab-2bc-2ac$$

Thus LHS = RHS

Since the LHS and RHS are equal, ~~can~~ the statement $(a+b-c)^2 = (a+b)^2+(a-c)^2+(b-c)^2-a^2-b^2-c^2$ is therefore true

Another attempt at Q1(i), this illustrates the kind of thing I am looking for, just a couple of sentences of explanation.

Assignment 1 feedback: Q2

Question 2a)

$x + \frac{1}{x} < 2$ where $x \in \mathbb{R}$ and $x \neq 0$

so $x + \frac{1}{x} - 2 < 0 \Rightarrow \frac{x^2 - 2x + 1}{x} < 0 \Rightarrow \frac{(x-1)^2}{x} < 0$

This means that the expression $\frac{(x-1)^2}{x}$ must be negative.
As the numerator is a perfect square, it will always be non-negative for any non-zero real value for x.
If the numerator will always be non-negative, for the fraction to be negative, the denominator x must be negative. It cannot be zero, as it will result in the fraction being undefined. Additionally, x is non-zero.
Hence, $x < 0$. $\square$

- I would add a word like "Suppose" at the start so it reads better.
- Note correct use of $\Longrightarrow$ to join logical propositions instead of to stand in for a word in the middle of a sentence.

Assignment 1 feedback: Q3

3)

b. $x^{m+n} = x^m \cdot x^n \ : P(n)$

Base case: let $n = 1$

$\Rightarrow x^{m+1} = x^m \cdot x^1$

$x^m \cdot x = x^m \cdot x$

$\therefore$ true for $p(n)$, $n = 1$

- Are you saying that $P(n)$ is the statement "$x^{m+n} = x^m x^n$? Say clearly what you mean. Use the examples from lectures and tutorials as a template for clear writing.
- What does the final line mean? What exactly is true? The reasoning is fine but remember I am looking for clear writing (audience: your average collegue in this course).
- (Also the case $n = 0$ is missed but this is a minor issue.)

Assignment 1 feedback: Q6

6) Assume the contrary that $z = x - y$ is rational. Then $z = a/b$ where $a, b \in \mathbb{Z}$ and $b \neq 0$.
Furthermore, since y is rational, $y = c/d$ where $c, d \in \mathbb{Z}$ and $d \neq 0$.

Therefore:

$$\frac{a}{b} = x - \frac{c}{d}$$

$$x = \frac{a}{b} + \frac{c}{d}$$

$$x = \frac{ad}{bd} + \frac{bc}{bd}$$

$$x = \frac{ad + bc}{bd}$$

As $a, b, c, d \in \mathbb{Z}$, ad is an integer and bc is an integer, then $ad + bc$ must also be an integer. Moreover, bd is also an integer where $bd \neq 0$. Hence the result $\frac{ad+bc}{bd}$ is rational, so x is rational.

This is a contradiction as it is stated above that x is irrational.
Consequentially $z = x - y$ must be irrational (the original statement is true). $\square$

- Essentially perfect!

MTH 1020 Week 5 tutorial

1. Get into groups of 3-4 people who all prepared a different question in advance.
2. Write your **preferred name** and **ID number** on the whiteboards so **I** can take attendance
3. Present your prepared question to each other as **I** come around, you should only take about 5min each for this.
4. Then get started on the other questions **in your groups**.
5. **At the end:** please erase the boards and return any markers etc that you used (you do not need to return the handouts)