Some properties of 2×2 matrices

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It has been known since at least the time of Poincaré that isometries of 3-dimensional hyperbolic space H^3 can be represented by 2×2-matrices over the complex numbers: the matrices represent fractional linear transformations on the sphere at infinity, and hyperbolic space is rigid enough that every hyperbolic motion is determined by such an action at infinity. A discrete subgroup of $\mathrm{SL}(2, \mathbb{C})$ is called a **Kleinian group**; the quotient of H^3 by the action of such a group is an orbifold.

A **link** is a disjoint union of knots. Consider a link embedded in $\mathbb{R}^3 \cup \{\infty\}$ and thickened to a solid tube. This tube can be removed, twisted around, and glued back in place; this process is called **Dehn surgery**. Every (closed, orientable, connected) 3-manifold can be obtained using this procedure by taking a suitable link and a suitable twist (this is the **Lickorish–Wallace theorem** of 1960/1962). A theorem of Thurston says that almost all the manifolds obtained via Dehn surgery are hyperbolic; in this sense, almost all 3-manifolds are hyperbolic [3, theorem 2.6]. Further, every complete hyperbolic 3-manifold may be realised as the quotient of H^3 by a Kleinian group (namely, its holonomy group). Since 3-manifolds are in general wild, Kleinian groups in general are wild and have fractal-generating actions.

Schottky groups form a special class of Kleinian group: these are groups generated by finitely many transformations defined by sending specified circles to specified circles. These groups are more well-behaved than the generic Kleinian group, but further specialisation allows rich structure to develop. Riley considered the particular case of torsion-free Schottky groups which are free on two generators, set up in such a way that the Kleinian manifold obtained in the quotient has boundary a four-times punctured sphere. These groups may be parameterised by a single complex parameter, and the domain of this parameter — the **Riley slice** — has been and continues to be a subject of much research into the deformation structure of the family of groups.

In this talk, we will describe the picture of the Riley slice as studied by Keen, Komori, and Series in [1, 2].

References

[1] Linda Keen and Caroline Series. "The Riley slice of Schottky space". In: *Proceedings of the London Mathematics Society* 3.1 (69 1994), pp. 72–90.

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